Bounded stability of the quiet standing posture: An intermittent control model

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Abstract

The paper presents a control model of body sway in quiet standing, which aims at achieving bounded stability by means of an intermittent control mechanism. Control bursts are generated when the current state vector exits an area of uncertainty around the reference point in the phase plane. This area is determined by the limited resolution of proprioceptive signals and the burst generation mechanism is predictive in the sense that it incorporates a rough, but working knowledge (internal model) of the biomechanics of the human inverted pendulum. We show that such a model, in spite of its simplicity and of the fact that it relies on very noisy measurements, is robust and can explain in a detailed way the measured sway patterns.

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1. Introduction

The human upright posture against gravity during quiet stance is maintained mostly by the ankle torque. In spite of the mechanical instability of the standing body, the posture is stable, but is it asymptotically stable? Asymptotic stability is indeed a very strong form of stability: it implies that the controller (including the actuators) can enforce a continuous, converging force field, attracting the body to an equilibrium state. Two possible implementation mechanisms of the attractive force field can be formulated: (1) actuator stiffness and (2) positional feedback (supplemented by derivative and/or integral components). The stiffness control hypothesis was proposed by Winter, Patla, Riedtyk, Ishac, and Winter (2001), but direct measurements of ankle stiffness (Casadio, Morasso, & Sanguineti, 2005; Loram & Lakie, 2002) failed to find sufficient levels of stiffness, which in fact ranges between 60% and 90% of the critical value determined by the destabilizing gravity torque. Moreover, EMG recordings of the ankle muscles during postural sway (Gatev, Thomas, Kepple, & Hallet, 1999; Loram, Maganaris, & Lakie, 2005; Nomura, Nakamura, Fukada, & Sakoda, 2007) do not provide any hint of a robust coactivation of muscles that, in theory, could induce a sufficient degree of stiffness and recent measurements of the contractile to series elastic stiffness ratio of the calf muscles (Loram, Maganaris, & Lakie, 2007) rule out even this possible mechanism of stiffness control, because ankle stiffness appears to be mainly determined by the compliant tendon: as a consequence, coactivation of the stiffer muscles would only have a minor effect on the ankle stiffness. The alternative control mechanism for achieving asymptotic stability, either in the form of a PD (proportional + derivative) controller (Masani, Vette, & Popovic, 2006) or a PID (proportional + integral + derivative) controller (Peterka, 2000, 2002) runs into other types of difficulties: although feedback gain parameters can be identified that stabilize the system, the stability margin is quite narrow, if we consider the large delays induced by segmental and supra-segmental loops. Moreover, since the stability achieved by a PD or PID controller can only be asymptotic, the residual sway movements are totally noise-driven and must be attributed to neural noise sources, in the sensory and/or motor part of the system.

However, asymptotic stability is not the only form of stability. A weaker form is bounded stability, where the control actions confine the system state in a small neighborhood of the equilibrium state, although are unable to keep it there. This kind of stability can be obtained by an intermittent controller, where stabilizing control actions occur in bursts and are event-driven. The control scheme is robust but even in the absence of noise is unable to damp out sway movements that remain persistent within a confined area. Thus, in this framework persistent sway patterns are not noise-driven but are the result of a limited-resolution control mechanism.

The intermittent nature of the control actions has been suggested by the analysis of posturographic patterns (Collins & De Luca, 1993; Jacono, Casadio, Morasso, & Sanguineti, 2004; Morasso & Sanguineti, 2002), EMG signals (Gatev et al., 1999; Loram & Lakie, 2002; Nomura et al., 2007), and the sway trajectories of the center of mass in the quasi-phase plane (Bottaro, Casadio, Morasso, & Sanguineti, 2005).

It is relatively easy and straightforward to formulate and simulate a continuous time postural control model based on linear feedback, driven by suitable noise sources; the model parameters can be tuned in such a way to reproduce many features of natural sway patterns, as shown by previous studies (Masani et al., 2006; Maurer, Mergner, & Peterka,

On the contrary, little effort has been put in the design and experimental validation of intermittent, non-linear control models, with the exception of a recent paper by van der Kooij and de Vlugt (2007). In this paper, we describe a candidate model that can stabilize a human-like inverted pendulum without enforcing asymptotic stability either by means of a very high intrinsic ankle stiffness or a strong and precise continuous sway feedback. The model is very robust because it can operate successfully with large delays and very low resolution afferent signals. We analyze the sway movements generated by the model and we compare them with natural sway, by using a number of posturographic indicators. It appears that the model captures the subtle structure of postural oscillations, including features that cannot be reproduced by continuous time, linear control, namely the bimodal distribution of sway patterns and the on–off structure of the gravity-related diagram, which displays the time course of the instantaneous correlation between sway and fall. The parameters of the proposed model were derived from the analysis of experimental sway patterns not from an optimization method. The comparison with PD-like models is carried out in qualitative and quantitative terms. In the latter case, we consider a specific PD model: the model proposed by Masani et al. (2006), who carried out a thorough sensitivity analysis and identified regions of stability in parametric space. In order to have a fair comparison we chose a set of parameters that fall in the middle of the region of stability.¹

We wish to emphasize the fact that our analysis is restricted to quiet standing. Postural stabilization in response to external or self-generated disturbances (anticipatory postural adjustments) as well as voluntary sway shifts requires additional control levels. The proposed intermittent stabilization mechanism is intended to be only the innermost control layer.

2. Methods

2.1. Experimental data

The experimental data on sway during quiet standing were collected in three different labs:

A. Center of Bioengineering of the Hospital La Colletta, Arenzano, Italy.
B. Department of Mechanical Science and Bioengineering of the Osaka University, Japan.
C. Center of Bioengineering, Foundation Don Gnocchi, Hospital S. Bartolomeo, Sarzana, Italy.

Ten young, healthy people were analyzed in each lab. All subjects gave their informed consent to participate.

Posturographic signals were recorded at 100 Hz with open and closed eyes, with a duration ranging from 40 s to 120 s. The participants were asked to stand quietly on the force

¹ The simulations with the Masani et al. (2006) model were carried out with the following parameters: $m = 76$ kg; $h = 0.87$ m; $J = 66$ kg m$^2$; $K_p = 1100$ Nm/rad; $K_d = 500$ Nm/rad/s; $\tau_f = 40$ ms; $\tau_e = 10$ ms; $\tau_m = 130$ ms.
platform, barefoot, with their arms relaxed on either side of their body, feet abducted at 20 deg, heels separated by about 2 cm. The following force platforms were used: RGM (Italy) in lab A, AMTI (USA) in lab B, Kistler (Switzerland) in lab C.

In this paper we focus on the AP (antero–posterior) component of the sway movements: $u$ refers to the CoP (center of pressure) position and $y$ to the corresponding position of the CoM (center of mass), with respect to the ankle: see Fig. 1, left panel.

The CoM time series $\{y(t), t = t_0 - t_f\}$ is obtained from the CoP time series $\{u(t), t = t_0 - t_f\}$ by using the following filter (Jacono et al., 2004)

$$Y(j\omega) = \frac{g/h}{\omega^2 + g/h} U(j\omega)$$

where $g$ is the acceleration of gravity and $h_e$ is the distance between the ankle joint and the CoM, modified by a “shape factor” that takes into account the distribution of body mass along the body axis ($h_e = 1.15 h$ as used by Bottaro et al. (2005)).

Since the sway amplitude is small, the sway angle of the body $\vartheta$ can be well approximated by the corresponding sine function: $\vartheta \approx \sin \vartheta = y/h$.

The oscillations of the CoM were detrended, as explained in a previous paper (Bottaro et al., 2005), by low-pass filtering the CoM trace, with a cut-off frequency of 0.01 Hz, and subtracting it from the original signal. This frequency was chosen in such a way to have an average reduction of the sway size of less than 5%.

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Fig. 1. Left panel – Inverted pendulum mechanics in the AP plane; $\vartheta$: sway angle; GRF: ground reaction force; $T_a$: ankle torque; $y$: position of the CoM (with respect to the ankle); $u$: position of the CoP (with respect to the ankle). Right panel – Postural control model; $\vartheta_{ref}, \vartheta, \dot{\vartheta}, \ddot{\vartheta}$: reference and actual sway angle, sway angular velocity; $T$: total ankle torque; $T_a$: ankle muscle torque; $T_{tonic}$: tonic control torque; $T_{phasic}$: phasic control torque; $T_m$: torque due to the mechanical muscle impedance; $T_g$: toppling gravity torque; $T_n$: noise torque. Afferent delay = efferent delay = 90 ms. Filter: low-pass with a cut-off frequency of 3 Hz.
2.2. The control model

The simulation studies described in this paper use the basic model sketched in Fig. 1 (left panel). The inverted pendulum, which represents the unstable “plant”, is characterized by the following equation:

\[ J \ddot{\theta} = T_g + T \]  

(2)

where \( m \) is the mass of the body (minus the feet), \( J \) is the corresponding moment of inertia with respect to the ankle joint, \( T_g = mgh\dot{\theta} \) is the toppling gravitational torque, and \( T \) is the total torque acting on the ankle due to different internal sources. This torque is decomposed into two main components: the ankle muscle torque \( T_a \) and the noise torque \( T_n \).

\( T_n \) is a stochastic component determined by self-generated disturbances like respiration or hemodynamics. It has a very low level: Conforto, Schmid, Camomilla, D’Alessio, and Cappozzo (2001) estimated that it does not exceed 0.2 N. We modeled it with a filtered white noise (cutoff frequency 4 Hz) and a gain factor chosen in such a way to have a RMS value of 0.2 Nm. The cutoff frequency was determined by taking into account that the frequency band of the CoP does not exceed 3–5 Hz.

\( T_a \) is decomposed into three contributions: \( T_a = T_m + T_{tonic} + T_{phasic} \).

1. \( T_m \) is the torque due to the mechanical impedance of the ankle muscles, which is characterized by a stiffness coefficient, \( K_a \), and a viscosity coefficient, \( B_a \):

\[ T_m = -K_a (\dot{\theta} - \dot{\theta}_{ref}) - B_a \ddot{\theta} \]

According to the direct measurement of ankle impedance performed by Casadio et al. (2005), we set \( K_a \) to be a fraction of the critical value of stiffness, which is the rate of growth of the toppling torque due to gravity: \( K_a = 0.7mg \).

The viscosity coefficient was set equal to a suitable small value: \( B_a = 3.71 \) Nm/rad/s.

2. \( T_{tonic} \) is the torque generated by the tonic motor controller, related to the reference sway angle, \( \dot{\theta}_{ref} \).

3. \( T_{phasic} \) is the torque generated by the intermittent controller.

\( T_g, T_{tonic}, T_m, T_n \) can be can measured or estimated. Therefore, it is possible to recover the phasic control torque according to the following equation:

\[ T_{phasic} = J \ddot{\theta} - T_g - T_{tonic} - T_m - T_n \]  

(3)

The tonic and phasic control mechanisms are affected by afferent and efferent delays. For both delays we used in the simulation model the figure of 90 ms, which is typically used in continuous time linear feedback models, with a total loop delay of \( \Delta T = 180 \) ms. An overall figure of 150–200 ms for the loop delay is supported by platform perturbation experiments, either using randomized sequences of small step-like disturbances (Casadio et al., 2005) or pseudo-random band-limited disturbances (Peterka, 2002). Of this delay, about 40 ms can be attributed to neural transmission time and neural preprocessing, as supported by the study of changes in muscle and cutaneous cerebral potentials during standing (Applegate, Gandevia, & Burke, 1988); about the same delay can be attributed to the efferent transmission of the command signals, the electromechanical delay in muscle contraction (Winter & Brookes, 1991), and the relaxation time constants of the viscous-elastic components of the muscles (Hof,
1998). The remaining 70–120 ms can be attributed to neural processing and for the sake of the simulations we split this figure in two equal parts for the afferent and efferent components, respectively: 50 ms + 50 ms.

2.2.1. The tonic controller

The purpose of the tonic controller is to specify a reference tilt angle, leaving to the phasic controller the task of constraining the sway movements to a small neighborhood of this angle. We implemented it as a simple torque amplifier, whose centrally generated input is $\dot{\theta}_{\text{ref}}$ and whose gain is $-mg\theta$. Therefore, the equation of the controller is as follows: $T_{\text{tonic}} = -mg\theta\dot{\theta}_{\text{ref}}$. The reference angle is constant or very slowly changing under voluntary control. The control mechanism that generates this control variable is outside the scope of the present paper.

2.2.2. The phasic controller

The proposed phasic controller is a modified version of a simple bang–bang controller: in bang–bang control a threshold element with a dead zone is applied to the difference between the current and the reference angles and then triggers a pulse of suitable amplitude (neither too small, nor too big), which induces the sway angle to oscillate in a quasi-sinusoidal manner around the reference angle. A control mechanism of this kind is very robust and quite insensitive to input and output noise. However, we know that given the overall range of sway angles (about 1 deg) the individual sways (from one peak of the sway curve to the next one) are much smaller and thus we need to improve this basic design in order to make it compatible with the experimental data.

We modelled the output of the phasic controller as a sequence of rectangular torque commands, with constant duration and variable amplitude, smoothed by a low-pass filter, with a cutoff frequency chosen by taking into account that the frequency band of the CoP does not exceed 3–5 Hz. In the model simulations we used the following second order filter: $F(s) = \frac{\Omega^2}{s^2 + 2\zeta\Omega s + \Omega^2}$, with $\Omega = 2\pi \cdot 3$ rad/s and $\zeta = 0.7$.

The phasic controller (Fig. 1, right panel) operates in an asynchronous, intermittent manner and is characterized by two components:

- A “sensory” component, which has the purpose of detecting the critical events for triggering the generation of stabilizing torque bursts and thus is in charge of the “when” factor;
- A “motor” component, which is supposed to compute the amplitude of the torque impulses that are appropriate for recalling the sway angle back to its reference value and thus is in charge of the “how much” factor.

Both components are formulated as functions of the state of the mechanical plant $[\theta, \dot{\theta}]$. On this purpose, we analyzed the behavior of the controlled system in the corresponding quasi-phase plane $[\theta - \theta_{\text{ref}}, \dot{\theta}]$. For the intermittent controller, the origin in this plane ($\theta = \theta_{\text{ref}}, \dot{\theta} = 0$) is an unstable equilibrium state when the control output is in the off state; the control bursts are generated when a suitable triggering condition is met, in order to push the system in the direction of the equilibrium state. The instability of the equilibrium state is likely to be reflected in the shape of the probability distribution of the sway angle, which is expected to have a local minimum in the origin (bimodal distribution). On the
contrary, for control models characterized by asymptotic stability the origin in the phase plane is a stable equilibrium state and we expect the distribution of sway angles to be unimodal, with a peak in the origin.

Another feature of sway movements that we need to reproduce by means of the intermittent control model comes from a previous study of sway movements (Bottaro et al., 2005): we found that sway movements are characterized by an alternation of two types of patterns: (1) a succession of several “small sways” (typically 4–6), which occur on the same side of the reference position, (2) two or more subsequent “large” sways, which cross the reference position. We called the former patterns A-type and the latter patterns B-type. We may consider the A-type patterns as manifestations of quasi-periodic attractors of the intermittent controller, respectively in the right and the left sides of the phase plane. In this framework, the B-type patterns correspond to phase transitions from one regime to the other.

For formulating the sensory and motor parts of the phasic controller, we carried out a first-order approximation of the probability distribution of the experimental data in the phase plane. The burst triggering signal was then generated in accordance with such distribution, with a refractory time equal to the duration of the bursts. The duration of the bursts was set to 250 ± 25 ms, considering that the figure of 3–4 bursts per second is supported by different studies (Bottaro et al., 2005; Loram et al., 2005; Loram, Gawthrop, & Lakie, 2006).

In order to identify the computational mechanism that determines the amplitude of the control bursts, we carried out an analysis of the experimental data in the phase plane in view of the following biomechanical law: a torque impulse (time integral of the torque burst) determines an equivalent variation of the angular momentum of the body inverted pendulum. Since the phasic controller is supposed to provide stabilizing commands, in the neighborhood of the reference sway angle, the initiation of the bursts can be identified in the following way:

- by local maxima of the speed profile of the sway angle, if the state is in the upper part of the phase plane (i.e., the body is swaying forward);
- by local minima of the speed profile of the sway angle, if the state is in the lower part of the phase plane (i.e., the body is swaying backward).

The termination time of the bursts is identified by the corresponding extremal point in the speed profile:

- a local minimum after a local maximum;
- a local maximum after a local minimum.

The amplitude of the torque impulses is then evaluated by multiplying the moment of inertia with the variation of the angular velocity and dividing the result by the impulse duration.

We expect of a successful phasic controller that it behaves in the following way:

- if the initial state is in the first quadrant of the phase plane (the body is tilted beyond the reference angle and is swaying forward) the impulse will be negative, in order to abort a potential forward fall;
if the initial state is in the third quadrant (the body is tilted behind the reference angle and is swaying backward) the impulse will be positive, in order to abort a potential backward fall.

We also expect the impulses to be higher, the greater the distance from the origin at the triggering time.

If impulses are triggered in the two other quadrants we expect the amplitudes to be smaller. The shape of the impulses is not relevant: we chose the rectangular shape for simplicity. What matters is their area, according to the biomechanical concept recalled above. In any case, the sequence of impulses is filtered with a low pass filter, as previously explained.

The intermittent control model was implemented in Simulink® and the simulation studies were carried out by using a variable-step integration method (ODE45).

2.3. Performance evaluation

The comparison between the performance of the model and the experimental sway patterns took into account a number of indicators/descriptors, which are listed in the following.

Indicators related to the CoM – \( y(t) \)

1. The RMS values of the \( y(t) \), \( \dot{y}(t) \), \( \ddot{y}(t) \) curves. In particular, the first one expresses the overall size of the sway movements.

2. The average duration of “large/main sways” \( \Delta T_y \), “small sways” \( \Delta T_{\dot{y}} \), and “torque bursts” \( \Delta T_\varepsilon \). The duration of large sways is computed as difference between successive zero crossings of the time-profile of \( y(t) - y_{\text{ref}} \), where \( y_{\text{ref}} = \theta_{\text{ref}} h \). The duration of small sways is computed as difference between successive zero crossings of the \( \dot{y}(t) \) curve. The duration of torque bursts is computed as difference between successive zero crossings of the \( \varepsilon(t) \) curve. In particular, we should consider that the biomechanics of the human inverted pendulum states that the acceleration of the CoM is proportional to the CoM – CoP difference, therefore the duration of the torque pulses is the same if computed on the CoM – CoP difference, i.e., by taking into account the time instants in which the CoP crosses the CoM up or down.

3. The histogram of the \( y(t) \) curve around its reference value \( y_{\text{ref}} \). The relevant feature is whether it is unimodal, with a peak in the reference position, or bimodal, with a local minimum in that position. Unimodality is indicative of asymptotic stability, with the origin as a point attractor, whereas bimodality is compatible with bounded stability and intermittent stabilization bursts around an unstable equilibrium point.

4. A qualitative/morphologic analysis of the trajectories in the quasi-phase plane: \( \{ y - y_{\text{ref}}, \dot{y} \} \) or \( \{ \theta - \theta_{\text{ref}}, \dot{\theta} \} \).

Indicators related to the CoP

5. The RMS value of the CoP.

6. The power spectral density (PSD) of the CoP, plotted in the log–log scale, in relation with the experimental finding (Collins & De Luca, 1993; van der Kooij & van der Helm, 2005) that such plot is piece-wise linear, with a smaller slope in the low-frequency range (<1 Hz) and a bigger slope in the high-frequency range (>1 Hz).
Indicators related to the phasic torque

7. The RMS of the phasic torque.

For the experimental data the phasic torque was computed by means of Eq. (3). In the simulation model it was obtained from the filtered impulse generator with additive noise.

3. Results

3.1. Evaluation of model parameters

As explained in the previous section, all parameters of the intermittent control model were selected according to published data or biomechanical considerations, except for the burst generation mechanism, namely the burst triggering signal and the computation of the burst amplitude. For both we need to analyze the experimental sway data: we carried out a first-order approximation of the probability distribution of the state vector \( x = [\dot{\theta}, \ddot{\theta}] \) for the population of subjects and, in particular, we computed the covariance matrix of \( x \)

\[
C = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \Rightarrow \begin{cases} \sigma_{\dot{\theta}} = \sqrt{c_{11}} = 0.0028 \text{ rad} \\ \sigma_{\ddot{\theta}} = \sqrt{c_{22}} = 0.0034 \text{ rad/s} \\ c_{12} = c_{21} \approx 0 \end{cases} \tag{4}
\]

The probability of generating a stabilizing torque burst, as a function of the current value of the state vector, must be consistent with the distribution of the state vector: it must be an increasing function of the distance of the current state from the equilibrium state \((\dot{\theta} = \dot{\theta}_{\text{ref}}; \ddot{\theta} = 0)\) and it must approach 1 when the state vector overcomes the ellipse, in the phase plane, whose main axes are, respectively, \(3 \cdot \sigma_{\dot{\theta}}\) and \(3 \cdot \sigma_{\ddot{\theta}}\). The experimental data above suggest that the off-diagonal elements of the \( C \) matrix can be neglected and thus the probability of generating a control burst can be approximated as follows

\[
P_{\text{burst}}(\dot{\theta}, \ddot{\theta}) = \text{erf} \left( \frac{\dot{\theta} - \dot{\theta}_{\text{ref}}}{\sqrt{2} \sigma_{\dot{\theta}}} \right) \cdot \text{erf} \left( \frac{\ddot{\theta}}{\sqrt{2} \sigma_{\ddot{\theta}}} \right) \tag{5}
\]

Fig. 2 shows the 3D plot of such probability distribution, taking into account that \( y = \dot{\theta} \), \( y_{\text{ref}} = \dot{\theta}_{\text{ref}} \), and \( \dot{y} = \ddot{\theta} \).

We introduced this probability in the simulation model by adopting the standard Monte Carlo approach:

- at each time instant in the simulation \( P_{\text{burst}} \) is computed from the current value of the state vector;
- a random number \( R \) is extracted, uniformly distributed between 0 and 1;
- if \( R < P_{\text{burst}} \) then the burst trigger is switched on.

The second analysis that we carried out on the experimental data, in order to characterize the burst generation mechanism, can be formulated as follows:

- Identification of the values of the state vector \([\dot{\theta}, \ddot{\theta}]\) at the time instants in which there is a local maximum of the speed profile of the sway angle, if the state is in the upper part of the phase plane, or a local minimum, if the state is in the lower part of the plane.
These time instants were assumed to mark the initiation $t_0$ of a burst. The termination $t_f$ of the burst is the time instant of the extremal point in the speed profile that follows in time.

- Computation of the speed variation $\Delta \dot{h} = \dot{h}(t_f) - \dot{h}(t_0)$ between the initial and final time of an identified burst. These variations, multiplied by the moment of inertia of the swaying body, were taken as a measure of the amplitude $A_{burst}$ of the torque impulses associated with the control bursts: $A_{burst} = \int_{t_0}^{t_f} T_{phasic}(t) \, dt = J\Delta \dot{h}$. The measurement unit of torque impulses is Nms.

A linear regression analysis of $\Delta \dot{h}$ onto $[\dot{\vartheta}, \dot{\vartheta}]$ was carried out by pooling all the data from the population of subjects

$$\Delta \dot{h} = p_0 + p_1 \dot{\vartheta} + p_2 \dot{\vartheta}$$ (6)

The estimated regression parameters were as follows: $p_0 = 0.001 \pm 0.00044$ rad/s; $p_1 = -0.86 \pm 0.0805$ s$^{-1}$; $p_2 = -0.79 \pm 0.0765$. The Pearson’s co-efficient of regression, $R^2$ was equal to .692. Therefore, we were confident to adopt the regression equation for the control burst generation mechanism

$$A_{burst} = J \cdot [p_0 + p_1 (\dot{\vartheta} + n_1) + p_2 (\dot{\vartheta} + n_2)]$$ (7)

The burst is generated at the time instant determined by Eq. (5) and with an amplitude that depends upon the angle (relative to the reference position) and the angular velocity at that time instant. The bias term $p_0$ is small as expected, because there is no reason to assume that the stabilization mechanism is not symmetric. The other two terms are negative as must be, because the control bursts must indeed counteract the impending fall. For simplicity, in the simulations we used the following values: $p_0 = 0$; $p_1 = -0.86$; $p_2 = -0.79$.

The burst equation also takes into account that the ranges of variations of the two state variables $\vartheta$ and $\dot{\vartheta}$ during quiet stance are very close to the perceptual threshold and thus
the corresponding measurements are characterized by a low value of the signal to noise ratio, i.e., a low resolution. This is expressed by means of the two noise terms \( n_1 \), \( n_2 \) whose RMS values are of the same order of magnitude of the RMS values of the two state variables, respectively. It must be emphasized that these noise terms are not crucial for the generation of the sway patterns, which are driven by the threshold mechanism expressed by Eq. (5), but simply help improving the fitting. In the simulations, \( n_1 \), \( n_2 \) are implemented by means of the Simulink\textsuperscript{®} “band-limited white noise” block, which generates normally distributed random numbers that are suitable for use in continuous or hybrid systems.

3.2. Simulation of the model

At this point the model is fully parameterized and we can simulate it, in order to test its stability and compare its performance with the experimental sway patterns. Table 1 summarizes the parameters used in the model equations and the corresponding reported simulations. It can be noted, in particular, that the critical value of ankle stiffness \( (mgh) \) is 502 Nm/rad and that summing ankle stiffness and the phasic torque component \( (K_a + J_p) \) we get a value that is still smaller than the critical stiffness.

The bursts generated by the phasic controller tend to occur in clusters of 3–4. The average duty cycle of this controller is about 50%. This means that during the simulated sway about half time is spent in falling and about half time in recovering the equilibrium state.

Fig. 3 shows typical patterns generated by the model as regards the time course of the CoM and CoP curves (top panel), the trajectories of the CoM in the phase plane (middle panel) and the PSD (Power spectral density) of the CoP signal over the usual observation time window of the posturographic examination (40 s). These patterns are qualitatively very similar to the experimental posturographic patterns reported in the literature. In particular, the PSD analysis exhibits the typical piece-wise linear profile that is described by many authors. In more quantitative terms, Table 2 compares several performance indicators or parameters in the model and in the population of subjects and they appear to be consistent: the data reported in the Table 2 are polled from the three labs after carrying out an ANOVA to make sure that ‘lab’ was not a significant factor.

A better fit could be obtained by a formal optimization process but optimal data fitting was not the purpose of this paper. The goal was to show that the intermittent control model, with model parameters derived from a variety of experimental evaluations, was indeed able to capture the overall features of posturographic data.

3.3. Indicators of intermittent control

The intermittent nature of the control process cannot be detected by global descriptors of the sway patterns, like the PSD of the CoP, because they cannot distinguish between asymptotic and bounded stability. However, the bounded stability assured by the proposed intermittent burst controller can be detected by taking into account the following facts:

1. The behavior of the system is accounted for by the dynamics of a (falling) inverted pendulum, between one command burst cluster and the next one.
The reference sway angle of the controller is less likely to occur than displaced angular configurations that correspond to the firing of control bursts, since it is an unstable equilibrium configuration.

This was translated into graphical terms in the following manner:

1. Gravity-related diagram. It displays the time course of the scalar product between the actual direction of the sway trajectory in the phase plane and the direction determined...
Fig. 3. Typical patterns generated by the intermittent control model. Top panel: CoM and CoP curves over time ($y(t)$, $u(t)$, respectively). Middle panel: Trajectories in the phase plane: $\dot{y}$ (mm/s) vs. $y$ (mm). Bottom panel: Power spectral density (PSD) of the CoP signal. AU: arbitrary units.
by a purely falling movement, i.e., the flow lines of the gravity field.\(^2\) When the value is positive and close to +1, it means that the motion of the body is consistent with a gravity-driven fall, either forward or backward. When the value is negative and close to −1, it suggests that an impulsive command has propelled the body in the opposite direction. Fig. 4 (top panel) shows that the curve generated by the control model oscillates between +1 and −1 in a regular way, with sharp transitions between the two values and an average duration of each oscillation of 0.43 ± 0.12 s. A similar behavior was reported in a previous paper (Bottaro et al., 2005) as regards the sway patterns of humans, with an average duration of each phase of 0.40 ± 0.29 s. In the same paper it is also reported that, if we apply this analysis to a PID-like control model, we get a very irregular trace with an average duration that is an order of magnitude smaller: 0.02 ± 0.03 s.

\(^2\) The flow lines of the inverted pendulum are generated by the equation \(\ddot{y} = \ddot{\theta}/h \), where \(\ddot{\theta}\) is the fraction of the acceleration of gravity that remains after having subtracted from the gravity torque the torque due to ankle stiffness.

2. **Histogram of the CoM.** Fig. 4 (bottom panel) shows that it is bimodal and this is a consequence of the fact that, as shown in Fig. 2 (middle panel), the state vector prefers to stay either on the left or the right of the equilibrium state \((y = y_{ref}, \dot{y} = 0)\), circling a few times in one half of the phase plane (small quasi-periodic orbits) before shifting to the other half by means of a larger orbit. On the contrary, the histogram of the sway patterns generated by an asymptotically stable system is necessarily unimodal because the state of the system tends to cluster around the stable equilibrium configuration. On the other hand, the histogram of the CoM speed is bell-shaped, with a peak in the origin \((\dot{y} = 0)\) whether the control model is continuous or intermittent but this is not informative, because any mechanism that is able to keep up the body inverted pendulum will be characterized by a balanced/unimodal speed distribution. As regards the experimental sway movements, there is some variability in the histograms of rather short sequences. We used 60 s sequences and detrended the CoM traces as explained in the methods. The majority of the 30 sequences exhibited non-unimodal histograms: 23 of 30. The average histogram is shown in Fig. 5 and clearly is not unimodal. The Jarque-Bera test for goodness-of-fit to a normal distribution, applied to the pooled CoM sequences, was not passed at a .05 confidence level. The same, of course, occurred to the CoM sequences generated by the intermittent control model, provided that they were at least

<table>
<thead>
<tr>
<th>Sway indicators</th>
<th>Measurement units</th>
<th>Model</th>
<th>Real data (average from 30 subjects)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMS CoM</td>
<td>mm</td>
<td>2.55</td>
<td>3.39 ± 1.39</td>
</tr>
<tr>
<td>RMS CoM speed</td>
<td>mm/s</td>
<td>3.25</td>
<td>3.20 ± 0.73</td>
</tr>
<tr>
<td>RMS CoM acceleration</td>
<td>mm/s²</td>
<td>9.04</td>
<td>12.41 ± 4.19</td>
</tr>
<tr>
<td>(\Delta T_y)</td>
<td>s</td>
<td>4.20</td>
<td>4.07 ± 1.99</td>
</tr>
<tr>
<td>(\Delta T_y)</td>
<td>s</td>
<td>1.40</td>
<td>1.18 ± 0.22</td>
</tr>
<tr>
<td>(\Delta T_y)</td>
<td>s</td>
<td>0.61</td>
<td>0.64 ± 0.06</td>
</tr>
<tr>
<td>RMS CoP</td>
<td>mm</td>
<td>2.96</td>
<td>3.82 ± 1.37</td>
</tr>
<tr>
<td>RMS phasic torque</td>
<td>Nm</td>
<td>0.97</td>
<td>0.89 ± 0.30</td>
</tr>
</tbody>
</table>
a few minutes long: if only 60 s sequences were considered, in about 15–20% of the cases we found unimodal distributions. On the contrary, the continuous feedback control model generated CoM sequences that passed the test.

3.4. Stability/robustness of the intermittent control model

The stability of the intermittent control model can be evaluated in three ways: (1) observation of the model behavior over a long time; (2) analysis of the response to unpredicted disturbances; (3) analysis of the stability margin in the phase plane.

Fig. 4. Indicators of intermittent control evaluated in the control model. Top panel: Gravity-related diagram. It displays the time profile of the scalar product between the actual direction of the sway trajectory in the phase plane and the direction determined by a purely falling movement. When the value is positive and close to +1, it means that the motion of the body is consistent with a gravity-driven fall, either forward or backward. When the value is negative and close to −1, it suggests that an impulsive command has propelled the body in the opposite direction. The ordinates express the cosine of an angle and thus are dimensionless. Bottom panel: Histogram of CoM positions around the reference position \( y = y_{\text{ref}} \). The bins are 1 mm wide. Ordinate: relative frequency. Note that the graphs are not unimodal but have a local minimum in the nominal position.
As regards the first point, we did not observe any fall of the model over several simulation runs of 10 min or more.

As regards the behavior of the control in the case of large, unpredicted disturbances, Fig. 6 shows the response of the system to a very sharp and large push forward (5 times greater than the RMS value of the phasic torque). It can be seen that although it shifts the CoM forward in a substantial way, the stable sway pattern is quickly recovered in a damped way, i.e., after 1–2 oscillations. For even larger disturbances the CoM could be shifted outside the support base, thus causing a “biomechanical destabilization” and a consequent change in the response type, typically a small step forward, but this is outside the scope of the present model.

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As regards the range of stability of the proposed controller, we carried out a simplified stability analysis in the phase plane, with the purpose of understanding, in particular, to which extent its stochastic features are essential for stability. Two simplifications were considered:

1. Deterministic burst triggering mechanism, using a sharp decision boundary in the phase space, which is coherent with the fuzzy boundary implied by the covariance matrix in Eq. (4);
2. Ideal, unfiltered Dirac impulses instead of the rectangular control bursts: the delivery of a torque impulse $A\delta(t)$ determines an instantaneous step-like change of the sway speed $\Delta\dot{y} = JA$.

Fig. 7 shows the phase portrait of the inverted pendulum, with the corresponding flow lines, which are generated by the equation $\dot{y} = \dot{\gamma}h_{c}y$ (see Eq. (7)). The two thick lines that satisfy the equation $\dot{y} = \mp \sqrt{\dot{\gamma}h_{c}}$ separate the phase plane into four regions: the region on the right corresponds to a progressive forward fall; the region on the left to a progressive backward fall; the top and bottom regions correspond, respectively, to transitions from backward to forward fall and from forward to backward fall. The red ellipse is the hard decision boundary. Control burst triggering points are intersections between the ellipse and outward going flow lines, in the first quadrant for potential forward falls and in the third

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3 For interpretation of color in Fig. 7, the reader is referred to the web version of this article.
quadrant for potential backward falls, respectively. The intermittent control is designed in such a way to push the state of the system in the right ballpark when the safety boundary is overcome. The blue arrows in the figure represent instantaneous sway velocity variations determined by control impulses whose amplitude is computed according to Eq. (7), with the empirically estimated values of the two control parameters $p_1$ and $p_2$. The final states, determined by such family of control actions, are aligned approximately along a straight line in the phase plane that has a negative slope, but is less steep than the thick line in Fig. 7 with the two converging arrows. Consider for example the triggering points in the first quadrant. The bounded stability of the intermittent control scheme requires that control bursts must satisfy the two following requirements:

1. they must be large enough to overcome the horizontal dashed line, in order to invert the falling direction or at least temporarily stop the fall;
2. they must not be too large, in the sense of overcoming the thick line with the two converging arrows. If that line is overcome, then the sequence of postural oscillations will progressively escape the bounded region of stability.

The figure shows that the empirically determined control parameters allow the control system to operate approximately in the middle of the range of bounded stability associated with the ideal intermittent control. The sway patterns generated from this simplified/deterministic version of the intermittent control system are sinusoidal like (as in bang–bang control) and do not resemble much the natural sways. The issue here is stability of the mechanism and we can say that it is not determined by its stochastic features. On the contrary, the simulations carried out with the non-ideal, intermittent controller, affected by delays, stochastic triggering, and non-ideal pulse generations, confirm the robustness of the control scheme.

In particular, the robustness of the control was also tested by varying the two main control parameters ($p_1$ and $p_2$) over a range up to ±10%: in no case the control became unstable. Another robustness check was performed on the total delay of the control loop. In the proposed model it is 180 ms (90 ms in the afferent and 90 ms in the efferent pathways, respectively). Stability is maintained if the delay is increased by more than 30%. This is not the case with continuous time PD controller. In order to have a fair comparison, we can restrict the analysis to the Masani et al. (2006) paper, which addressed the issue of robustness of a PD controller for variable delays and variable proportional-derivative parameters. This study shows that the tolerated delay before instability becomes larger for smaller values of $K_p$ and $K_d$ but the range of stability also becomes smaller; in any case, the overall delay before instability never exceeds 185 ms.

4. Discussion

We have shown that a simple intermittent control model can explain the variability and, at the same time, the bounded stability of sway patterns in quiet standing. This is not the only example of biological intermittent feedback control. Another well known case, for instance, is the control mechanism of saccadic eye movements (Bahill, Clark, & Stark, 1975; Miller & Robins, 1992; Rodgers, Munoz, Scott, & Martin Paré, 2006). In both cases, in our opinion, the evolutionary pressure to choose the intermittent design instead of the apparently more straightforward continuous time feedback design
comes from the need to overcome the potential instability due to the propagation delays in the control loop. In the saccadic system, in which the mechanical plant is stable although quite laggy, stability of the feedback control could be achieved easily by means of suitably low values of the loop gain, but this would reduce the speed of the movements by at least an order of magnitude with respect to the observed values. The saccadic intermittent control, on the contrary, closes the loop only at specific time instants and at those instants triggers maximal bursts modulated in time (phasic commands) on top of the eye position-dependent tonic activity. Propagation delays do not affect stability but only determine a delay in the response delivery. In posture control the mechanical plant is unstable, to start with, and keeping the loop closed all the time makes the overall stability a real challenge. It is also worth mentioning that performing saccadic eye movements improves postural control (Rougier & Garin, 2007) as regards a reduction of the amplitude of the CoP and CoP–CoM signals, whereas simple blinking does not: considering that the average frequency of saccadic eye movements is quite similar to the estimated frequency of posturographic bursts in the intermittent control model, one might envisage some form of mutually beneficial entraining among the two control mechanisms, to be tested in suitable experimental paradigms.

The danger of continuous time feedback control, with substantial delays in the loop, is to inject too much energy in the loop at wrong time instants, with the problem of dissipating a lot of excess energy. A consequence is that continuous feedback control is potentially less energy efficient than intermittent impulsive control if operating close to instability conditions. This statement is confirmed by evaluating the power delivered by the controllers during sway in the two models. In order to make a fair comparison, let us be clear about the model parameters. The intermittent control model has the parameters listed in Table 1; for the continuous time feedback control we used the specific model proposed by Masani et al. (2006) and, as previously said, we chose a set of parameters that fell in the middle of the region of stability. Moreover, the noise power of the Masani et al. (2006) model was adapted in such a way to duplicate the RMS value of the CoM evaluated in the intermittent control model: 3.1 mm. The instantaneous power is the product of the angular speed and the torque delivered by the controller and we computed the RMS value over a 60 s time window in both cases. In the intermittent control model, the phasic controller delivers 5 mW and the tonic controller 39 mW, with a total of 44 mW. The PD-like controller delivers a much greater power: 143 mW. It may be argued that this large difference can be due to the fact that in the PD-like controller the proportional control element virtually incorporates ankle stiffness, thus yielding an overestimate of the delivered power. Therefore, we modified the Masani et al. (2006) model by introducing a stiffness elements (with the same stiffness parameter of the intermittent control model: $K_a = 0.7nh$) and we subtracted $K_a$ from $K_p$. In this case, the power delivered by the controller is reduced from 143 mW to 97 mW but still it is more than twice the power required by the intermittent controller. Therefore, we are confident to say that the intermittent controller is more energy efficient than the PD-like controller. The energetic aspect of the standing paradigm was also addressed by Lakie, Caplan, and Loram (2003). In particular, they focused on the role of the compliant ankle joint in the stabilization of posture: rather than providing passive stability, the intrinsic stiffness acts as an energy efficient buffer which is intermittently loaded/unloaded and thus provides decoupling between muscle and body.

As regards robustness, we have shown in the previous section that PD control is less robust than intermittent control as regards the sensitivity to variations in loop delay: a
10% increase is sufficient to make the loop unstable in comparison with a 33% increase necessary to destabilize the intermittent control. The fact is that continuous PD control is either asymptotically stable or openly unstable. A relevant consequence is that sway oscillations would naturally die out in absence of noise and thus the distribution of sway angles is bound to cluster around the equilibrium angle with a clear peak in the probability distribution. The experimental data, on the contrary, show a tendency to bimodal distribution. This is a consequence, in our opinion, of a control mechanism where the attractor of the system is not the equilibrium position but a bounded area around it. The gravity-related diagram documents in which manner the system’s state bounces back and forth during sway movements: about 50% of the times it is driven “downhill” by gravity, on one side or the other of the equilibrium configuration, and the rest of the time it is driven “uphill” by the command bursts. The defenders of PD-like control argue that the sway patterns generated by this class of models resemble very closely the experimental ones if one considers a number of sway measures (Maurer & Peterka, 2005; van der Kooij & de Vlugt, 2007). However, this is not sufficient because such traditional parameters do not address the type of stability assured by the controller, but global aspects, in the time or frequency domain, that can be reproduced equally well by an asymptotically stable control system and a control system characterized by bounded stability. In general, the bimodal distribution of sway angles is incompatible with sway patterns determined by an asymptotically stable model unless we hypothesize the presence of a very low-frequency noise source affecting the reference angle. This is the reason for which we computed the histograms of the sway patterns after they were detrended by subtracting the hidden equilibrium trajectory of the CoM: the tendency to a bimodal distribution remained as well the sharp transitions in the gravity-related diagram.

In the proposed model of postural stabilization during quiet standing the persistent sway patterns are not determined by additive noise sources but by the very low-resolution of proprioceptive signals, operated near their transduction threshold; this may explain in a natural way the apparent paradox that simply touching an external object while standing, even if the transmitted force is barely measurable, dramatically reduces the sway size, although it does not abolish it (Jeka & Lackner, 1995; Lackner et al., 1999). In fact, what is important in our opinion is not the contact force but the sensory information, associated with it, about the direction and the amount of sway: this information is likely to improve the resolution and accuracy of the estimate of the state vector. In the usual standing situation, such estimate is mainly given by proprioceptive signals originating in the ankle, ankle muscles, and foot plant. The small size of the sway angle does not allow such estimate to be accurate. However, the proprioceptive signal provided by the fingertip has the great advantage of occurring at a large distance from the rotation axis and thus it is “amplified” accordingly, improving the signal to noise ratio and thus the resolution of the measurement.

In principle, the intermittent control model can be applied to the medio-lateral component of sway as well. However, the biomechanical model is more complex (it is a closed, not an open kinematic chain); there is an additional parameter to take into account (the lateral displacement of the two feet); no estimates are available (as far as we know) of the ankle stiffness for medio-lateral movements. Therefore, the extension to medio-lateral movements is left to a future study.

Further extensions are related to the fact that quiet standing is a limit condition that rarely occurs in isolation during daily life activities. For example, the task of bipedal
Voluntary shifts of the CoM may occur in dance, when synchronizing the body motion with a sound source, or when compensating a predictable postural perturbation, such as the rolling motion of a boat. In such cases, the simplified inverted pendulum model is inadequate and multi-joint coordination must be addressed. The research in this area mostly focused on the issue of muscle or kinematic synergies (Alexandrov, Frolov, & Massion, 1998; Krishnamoorthy, Goodman, Zatsiorsky, & Latash, 2003) attempting to identify small sets of central control variables associated with the synergies; little work has been done for developing computational control models, with the exception of constrained optimization approaches (Tagliabue, Pedrocchi, Pozzo, & Ferrigno, 2007). The intermittent control model can be generalized to a multi-joint situation by implementing a bounded stability strategy for the CoM in a similar way to the model described in this paper. In the limiting case of voluntary shifts of the CoM in the sagittal plane, we can still use the inverted pendulum approximation and the controller may be extended by means of an additional control loop, nested around the controller of Fig. 1 and linked to the tonic component of the model. This additional control loop could well operate as a continuous feedback mechanism because most of the objections to this kind of design, which are relevant to the quiet standing condition, do not apply in this case: the resolution of the sensory afferents is much better and the stability issue, related to the delay in the control loop, is de-emphasized, because the intermittent control in the inner loop is masking the intrinsic instability of the inverted pendulum. A similar approach could also be applied for explaining the strong temporal coordination between postural and focal task components in whole body reaching (Patron, Stapley, & Pozzo, 2005).

Let us consider that the essence of the proposed control model is (1) an estimator of the CoM state, with low-resolution but sufficient to detect the falling direction, (2) a threshold element that delimits an area of bounded stability, and (3) a burst generator mechanism, roughly tuned as a result of training. We think that the same approach can be generalized to quite different control paradigms that rely on different sensory channels and/or different muscle groups. An example is standing with a very reduced support base, such as standing on stilts or standing on a tight rope. Stilt walking has been studied as regards energetics (Vaida, Anton-Kuchly, & Varene, 1981) not the control mechanisms. From the biomechanical point of view the main feature of stilt standing/walking is that the position of the CoP is constrained by the environment. However, the same dynamic equation that characterizes quiet standing (Morasso & Schieppati, 1999) still applies: the acceleration of the CoM is proportional to the CoM – CoP difference, i.e., $\ddot{y} \propto y - u$. In postural control paradigms with normal support conditions $u$ is the control variable and the control action is focused on the ankle joint (ankle strategy), relying mostly on proprioceptive information. In reduced/constrained support conditions $y$ becomes the control variable and the control action is spread to a number of joints of the lower and higher limb (hip or, more generally, whole body strategy), relying mostly on vestibular and visual information. However, the same control architecture can be adapted to both situations. A similar generalization is also possible as regards balancing a pole on a finger. The CoP of the artificial inverted pendulum is fixed with respect to the supporting finger but its position in space can be controlled by means of arm movements; moreover, it is a common experience that the range of motion of the supporting finger (the CoP of the pendulum) is larger than the range of motion of the CoM of the pendulum, as it happens in the CoM/CoP interplay of quiet standing. Vestibular information does not help in this case whereas vision
becomes dominant. Foo, Kelso, and de Guzman (2000) focused the attention on the sensory affordances inherent in this control paradigm; Mehta and Schaal (2002) suggested that the experimental data support the existence of a forward model in the sensory preprocessing loop of control. The intermittent control model can be adapted in a natural way to this paradigm, using different sensory channels for evaluating the relative position of the state of the pendulum within a successful bounded stability region (vision and touch instead of ankle/foot receptors) and different effectors (arm muscles instead of ankle muscles). The forward model proposed by Mehta and Schaal (2002) would play the role of burst generation mechanism, triggered when visual/touch thresholds are crossed.

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References


